ST. MARGARET SR. SEC. SCHOOL
MID TERM EXAMINATION 2024-25
MATHEMATICS (041)
CLASS XII

Time: 3Hr **SAMPLE PAPER.** **M.M: 80**

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and follow them:

i) Question paper is divided into 5 sections-Section A, B,C,D and E.
ii) In Section A- Question Number 1 to 18 are Multiple Choice Questions(MCQ) type and Question Number 19 to 20 are Assertion-Reason based questions of 1 mark each.
iii) In Section B-Question Number 21 to 25 are Very Short Answer(VSA) type questions of 2 marks.
iv) In Section C- Question Number 26 to 31 are Short Answer(SA) type questions carrying 3 marks each.
v) In Section D-Question Number 32 to 35 are Long Answer(LA) type questions carrying 5 marks each.
vi) In Section E-Question Number 36 to 38 are case study based questions carrying 4 marks each.
vii) There is an internal choice in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

SECTION- A

1. The number of equivalence relation in the set {1,2,3} containing the elements (1,2) and (2,1) is
   (a) 0 (b)1 (c) 2 (d) 3
2. $\int_{-1}^{1} e^{|x|}$ dx =
   (a) 2e-1 (b) 2e-2 (c) e^2-1 (d) e-2
3. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$ then value of a+b-c+2d is
   (a) 8 (b) 10 (c) 4 (d) -8
4. If $|A|$ = 2 where A is a 2x2 matrix then $|4A^{-1}|$ =
   (a) 4 (b) 2 (c) 8 (d) 32
5. The function f(x)= $[x]$, greatest integer function is continuous at
   (a_ 4 (b) -2 (c) 1 (d) 1.5
6. If y= $\sin^2(x^3)$ then dy/dx =
   (a) $2\sin x^3 \cos x^3$ (b) $3x^3\sin x^3\cos x^3$ (c) $6x^2\sin x^3\cos x^3$ (d) $2x^2\sin^2(x^{3)}$
7. If f(x)= a(x-cosx) is strictly decreasing in R then a belongs to
   (a) {0} (b) (0,∞) (c) (-∞ ,0) (d) $(-\infty, -\infty)$
8. The total cost C (x) in rupees associated with the production of x units of an item is given by C(x)= $0.005x^3$-$0.002x^2$+30x+5000 , then value of marginal cost when 3 units are produced is
   (a) 30.02 (b) 3.02 (c) 300.2 (d) 0.3002
9. $\int \frac{secx}{secx-tanx}$ dx =
   (a) Secx-tanx +C (b) secx+ tanx +C (c) tanx-secx +C (d) –secx-tanx+C
10. If f(x)= $\int_0^x tsint\, dt$ then f`(x)=
    (a) Cosx+ xsinx (b) xsinx (c) x cosx (d) sinx + x cosx .

11.The function f:R→ R defined by f(x)=4+3cosx is

(a) Bijective (b) one-one but not onto (c) onto but not one-one (d) neither one-one nor onto.

12. Let $\theta = \sin^{-1}(\sin(-600^0))$, then value of θ =

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$

13.The number of all possible matrices of order 3x3 with each entry 0 or 1 is

(a) 27 (b) 18 (c) 81 (d) 512

14. If x=-4 is aroot of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}$=0 then the sum of the other two roots is

(a) 4 (b) -3 (c) 2 (d) 5

15. the value of $\int e^x secx(1+\tan x)dx$ is

(a)$e^x cosx$ +C (b) $e^x secx$ +C (c) $e^x sinx$ +C (d) $e^x tanx$ +C

16. The value of $\int 2^x 3^x$ dx =

(a) $\frac{6^x}{log6}$ +C (b)5^xlog5+C (c) $\frac{5^x}{log5}$ +C (d) $\frac{3^x}{log3}$ +C

17. The value of $\cot(\cos^{-1}(\frac{7}{25)})$ is

(a) $\frac{25}{24}$ (b) $\frac{25}{7}$ (c) $\frac{24}{25}$ (d) $\frac{7}{24}$

18. If y= $\cos^{-1}$(sinx) then dy/dx=

(a) 1 (b) -1 (c) –sinx (d) cosx

Assertion-Reason Based Questions

In the following questions 19 and 20, a statement of Assertion(A) is followed by a statement of a Reason(R). Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false and (R) is true.

19. Assertion: $\sin^{-1}$ (sinx)= x for all x$\in [-\pi/2 , \pi/2]$

Reason: $\sin^{-1}$ (sin$7\pi/6$)= $7\pi/6$.

20. Assertion: The value of determinant A, A= $\begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}$ is zero.

Reason : A is invertible matrix.

SECTION- B

21. If A and B are symmetric matrices, such that AB and BA are both defined then prove that AB-BA is a skew symmetric matrix.

22. The side of an equilateral triangle increasing at the rate of 2cm/sec. At what rate its area is increasing when its edge is 12 cm.

23. Find the value of $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx}$ dx.

24. For what value of k is the function f(x)= { $\frac{sin5x}{3x}$ + cosx , if x≠ 0

K if x=0

Continuous at x=0.

25. Draw the graph of $\sin^{-1}$x .

SECTION-C

26. Check whether the relation R in R defined by R= { (a,b) : a≤ b^2 is reflexive , symmetric or transitive.

27. Find the absolute maximum and minimum valuesof f(x)=$\sin^2$x-cosx.x$\in$ [0, π].

OR

Find the values of x for which y= $\sin^4$x+$\cos^4$x is strictly increasing or decreasing function.

28. If A= $\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}$ then show that A^3-23A-40I= O .

29. $\int_{-\pi}^{\pi}(cosax - cosbx)^2$ dx.

30. $\int \frac{2x}{(x^2+1)(x^2+2)}$ dx OR Find : $\int[\log\log(x) + (\frac{1}{logx)^2})\ ]$ dx.

31. If x= sint and y= sinpt then prove that $(1-x^2)\ y_2 - xy_1 + p^2y = 0$.

OR

If x= asin2t (1+cos2t) and y= bcos2t(1-cos2t) then find dy/dx at x= $\frac{\pi}{4}$.

SECTION- D

32. $\int_0^{\frac{\pi}{2}} logsinxdx$ dx .

OR

Evaluate : $\int_0^1 tan^{-1}(\frac{2x-1}{1+x-x^2})$.

33. Show that the function f:R→R defined by $f(x)= 2x^3-7$ for all x∈ R is bijective.

34. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find its maximum volume.

35. If A = $\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$ find A^{-1} . Hence solve the system of equations:

x+y-z=3 , 2x+3y+z=10 and 3x-y-7z=1 .

OR

If A= $\begin{bmatrix}1 & tanx\\ -tanx & 1\end{bmatrix}$, show that $A'A^{-1}$= $\begin{bmatrix}cos2x & -sin2x\\ sin2xx & cos2x\end{bmatrix}$.

SECTION- E

36. A function f(x) is differentiable at a point c in its domain if $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ exists finitely. This limit, if it exists, is called the derivative of f(x) at x=c and is denoted by f'(c). Based on the above information answer the following:

(i) Check whether f(x)=|x+1| is differentiable at x=-1.

(ii) If f(3)=6 and f'(3)=2, then find $\lim_{x\to 3}\frac{xf(3)-3f(x)}{x-3}$.

OR

If f(x) is differentiable at x=c, then find $\lim_{x\to 3}\frac{xf(c)-cf(x)}{x-c}$.

37. Amit,Biraj and Chirag were given the task of creating a square matrix of order 2.

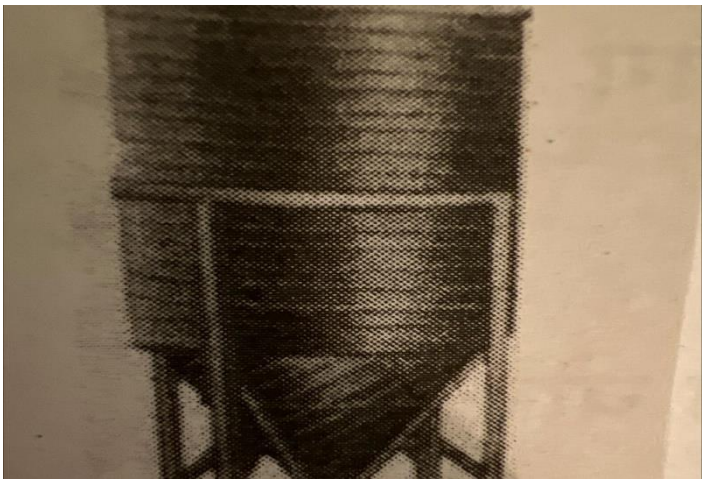
A=$\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}, B = \begin{bmatrix}4 & 0\\ 1 & 5\end{bmatrix}, C = \begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix}$ were created by Amit ,Biraj , Chirag.

(i) Find the sum of the matrices A,B,C ; A+(B+C).

(ii) Evaluate $(A^T)^T$.

(iii) Find the matrix AC-BC. OR Find the matrix (a+b)B when a=4,b=-2.

38. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water.

Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45°.
Answer the following questions:
(i)Find the volume of water in the tank in terms of its radius r.
(ii)Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm.
(iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm.

OR

Find the rate of change of height 'h' at an instant when slant height is 4 cm.